KFU's Interregional Subject Olympiad
profile "Mathematics" (final stage)
for 10-11th grade students
(March 2025)

Problem 1 (10 points). Peter bought several pens for $1.5. If he had bought pens for $1.2, he could have bought 3 more pens for the same money. If he had bought pens for $2.1, he could have bought 5 fewer pens for the same money. How many pens did Peter buy?

Problem 2 (15 points). The teacher is preparing tasks of the form "Find $GCD(a,b)$". As a and b, he takes two different divisors of the number 2025 (the order of the numbers in the pair does not matter).

a) How many different versions of such a task can he prepare?

b) How many of them has the answer $GCD(a,b)=1$?

Problem 3 (15 points). Let's say a triangle is circumscribed about a pentagon if each side of the triangle contains a side of the pentagon. Draw a convex pentagon about which it is possible to circumscribe a) exactly one triangle; b) exactly two triangles.

Problem 4 (20 points). Mrs. Smith buys boxes of treats, each containing one of the toys: either a doll or a robot. Before opening a box, she doesn't know which toy is inside, but she looks inside immediately after buying the box.

Mrs. Smith has two sons, and she wants to give each of them a robot. Dolls are three times more common than robots. What is the probability that she will have to buy a) exactly 3 boxes; b) at least 4 boxes?

Problem 5 (20 points). For some $a \neq 1$, parabolas with equations $y = ax^2 + x + 1$; $y = x^2 + x + a$ have a common tangent (the points of tangency do not have to coincide). Prove that this tangent passes through the origin.

Problem 6 (20 points). In a convex quadrilateral *ABCD*, the bisectors of the interior angles are drawn. The bisectors of angles *A* and *B* intersect at point *P*, the bisectors of angles *B* and *C* intersect at point *Q*, the bisectors of angles *C* and *D* intersect at point *R*, and the bisectors of angles *D* and *A* intersect at point *S*. The intersection points form the quadrilateral *PQRS*.

a) Prove that quadrilateral *PQRS* is cyclic.

b) Let *PQRS* be a square. Prove that *ABCD* is a rectangle.