Interregional subject Olympiads of KFU
profile "Physics"
final stage
2024-2025 academic year
10-11th grades
Solutions

Explanations of the Evaluation Criteria

To achieve a full score for each criterion, the solution must include a law, equation, inequality, idea, or technique that is specifically suitable for this task and is written correctly. If the solution contains general wording or errors on this point, a lower score, including zero points, will be given. Solutions that deviate significantly from the author's intentions will be evaluated outside of the criteria.

Problem 1. **(15 p.)** Two iron ($^{56}_{26}$Fe) balls of 1 mole of atoms each are located at a large distance from each other. Under the action of light falling on them each of the balls has lost approximately equal number of electrons. At a certain moment it turned out that the interaction force of the balls became equal to 0. Find the number of electrons N that each of the balls has lost at this moment. Neglect the influence of gravity of other bodies. Electric constant $k = 9\cdot10^9$ N·m^2/C^2, gravitational constant $G = 6.67\cdot10^{-11}$ N·m^2/kg^2, electron charge $e = -1.6\cdot10^{-19}$ C.

Possible solution:

The mass of the balls in this case is numerically equal to their molar mass. Write the balance of forces acting on each of the balls.

$$G\frac{M_{Fe}^2}{R^2} = k\frac{N^2e^2}{R^2}$$

Then we express the number of electrons

$$N^2 = \frac{G}{k}\frac{M_{Fe}^2}{e^2}$$

$$N = \frac{M_{Fe}}{e}\sqrt{\frac{G}{k}} \approx 3\cdot10^7$$

Evaluation criteria:

The mass of the balls is equal to their molar mass.	3
The gravitational force is correctly written.	3
Coulomb's law is correctly written.	3
Analytical expression for the number of electrons is correct (not required, if numerical answer is correct, then the answer is counted)	3
Numerical answer is correct.	3

Problem 2. (17 p.) A piece of tin was dropped without initial velocity from a height of 13500 m above the surface of a planet without atmosphere. The flight time to the surface of the planet is 73.5 s. Having fallen, it absolutely inelastically hit the surface of the planet. What part of the tin melted if 90% of the heat released on impact was absorbed by the tin? Before the impact, the temperature of the tin was 32 ˚C. The specific heat capacity of tin is 230 J/(kg·˚C), the specific heat of fusion of tin is 59 kJ/kg, and the melting point of tin is 232 ˚C. Consider that the radius of the planet is much greater than the height from which the piece of tin fell.

Possible solution:

Write the energy conservation law. In this case, since the impact was absolutely inelastic, it can be considered that all kinetic energy of the piece of tin E was transferred to thermal energy Q. Since there is no viscous friction of air, all potential energy of the piece of tin (counted from the planet surface), which was at the height H = 13500 m, transformed into kinetic energy. Then we can write

$$E = Q$$

Introduce the notations: m - mass of a piece of tin, g - free fall acceleration, c - specific heat capacity of tin, λ - specific heat of melting of tin, ΔT = 200 ˚C - change of temperature of tin from initial T_1 = 32 ˚C to melting temperature T_2 = 232 ˚C, t - time of fall, α - required molten mass fraction of tin.

Then potential energy of a piece of tin before falling

$$E = mgH$$

$$g = \frac{2H}{t^2} = 5\ m/s^2$$

90% (i.e. 0.9) of the heat released was used to heat it to the melting point and the phase transition

$$0.9Q = mc\Delta T + \alpha m\lambda$$

Substituting the first two equations into the fourth one and reducing the mass, we get

$$0.9gH = c\Delta T + \alpha\lambda$$

Hence the required molten part of tin is

$$\alpha = \frac{0.9gH - c\Delta T}{\lambda} = 0.25 = 25\%.$$

Evaluation criteria:

Thermal energy is equal to kinetic energy or change in potential energy.	4
The acceleration of free fall is found.	3
Heat balance equation.	4
Analytical expression for the fraction of tin that has melted (not required, if numerical answer is correct, then the answer is counted).	3
Numerical answer is correct.	3

Problem 3. (17 p.) The two flat plates of each 3 cm thick of porous materials are firmly connected with each other by their large edges. The density of the first and second materials in the dry state is the same, but each cubic centimeter of the material of the underwater part of the first plate absorbs 2 times more water than the material of the second plate. The absorption of water by the above-water part of the plates is neglected. If the system is floated in water so that the first material is at the bottom, the equilibrium immersion depth is 4 cm. If the structure is inverted, after a long time the immersion depth will be 2 cm. Find the material densities of the plates in the dry state. The density of water is 1000 kg/m^3. The process of water absorption is completely reversible. The width and length of the plates are much larger than the thickness, and the system in both cases floats so that the plane of the plates is horizontal.

Possible solution:

Write the balance of forces acting on a floating body in two cases. Equate the weight of the body and Archimedes' force. The immersion depths in the first and second cases are denoted by h_1 and h_2, respectively, the thickness of the plate by d. The mass of water absorbed by 1 m^3 of the material of the second plate is denoted by a. Denote the density of water as ρ_w and the required density as ρ:

$$2d\rho + 2ad + (h_1 - d)a = 2d\rho + ad + ah_1 = \rho_w h_1$$

$$2d\rho + ah_2 = \rho_w h_2$$

Here we have reduced the free fall acceleration and the area of the plates. After subtracting the second equation from the first one, we get a

$$a = \rho_w \left(\frac{h_1 - h_2}{h_1 - h_2 + d} \right) = 400 \text{ kg/m}^3$$

Then we find ρ from the second equation

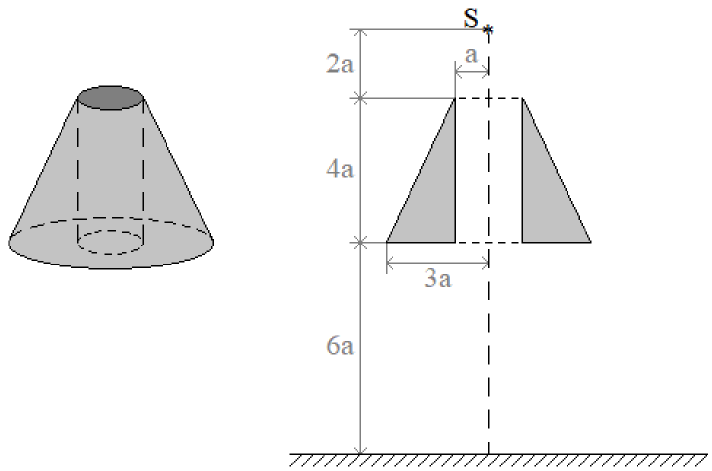
$$\rho = \frac{h_2(\rho_w - a)}{2d} = 200 \text{ kg/m}^3$$

Evaluation criteria:

Body weight is expressed via densities in two cases.	4
Archimedes' force is written for two cases.	4
The vertical force balance is written for two cases.	4
Analytical expression for the density of the plates is obtained (not required, if the numerical answer is correct, then the answer is counted)	3
Numerical value of the density of the plates is correct.	2

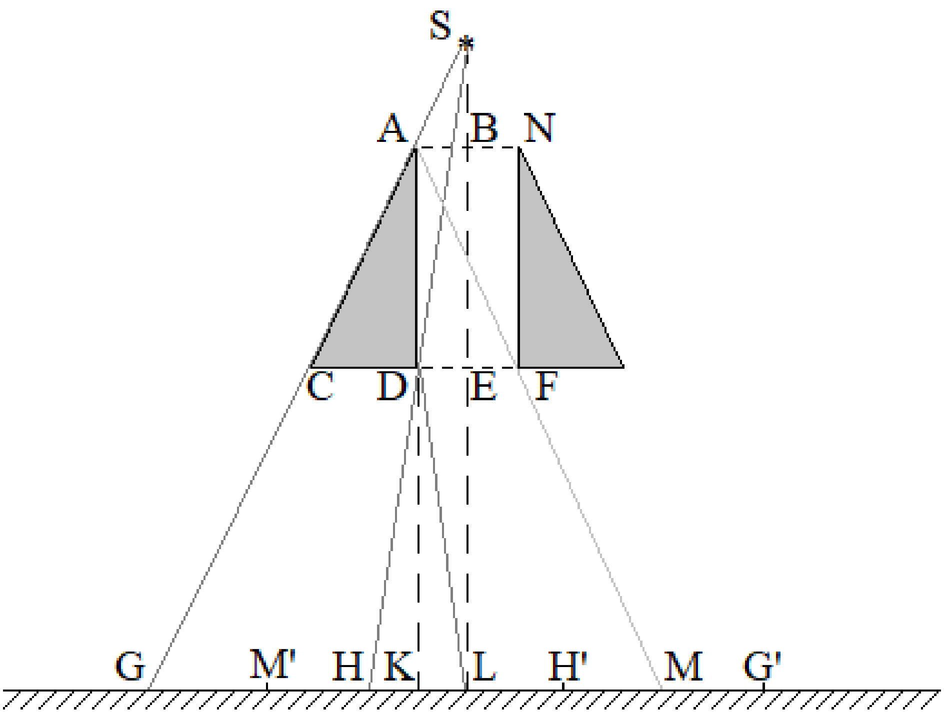
Problem 4. (17 p.) The body shown in the figure on the left is a truncated straight circular cone with a through cylindrical hole, one of the bases of which coincides with the smaller base of the cone. The cone is positioned over a horizontal surface such that its bases are parallel to the surface. On the axis of the cone there is a point light source S, as shown in the figure on the right

(here the cone in question is shown in section). The right figure also shows the dimensions and distances in units of a, where $a = \frac{2}{\sqrt{\pi}}$ cm. The cone is made of an opaque material and the surface of the walls of the hole inside the cone is mirror-like. Find the area of the shadow cast by the cone on a horizontal surface.



Possible solution:

In order to determine the areas of the horizontal surface illuminated by light, consider the path of rays in the section of the cone nearby the extreme points.



The ray going through point A, due to the similarity of triangles ABS and CES (by the two sides CE/AB=3a/a=ES/BS=(4a+2a)/2a=3 and the angle between them ∠ABS = ∠CES = 90˚), will "slide" on the surface of the cone and will get to point G. The distance of point G from the axis of the cone, based on the similarity of triangles ABS and GLS (by two sides GL/AB=GL/a=LS/BS=(6a+4a+2a)/2a=6 and the angle between them ∠ABS=∠CES=90˚),

$$LG = 6a$$

The surface area to the left of point G will be illuminated. The ray coming “just to the right” of point A will hit the mirror surface inside the hole and will be reflected from it at an angle to the perpendicular, ∠NAF = ∠BAS. Then the reflected ray will go near the point F, which coincides with the lower corner of the opposite mirror surface, because triangles NAF and BAS are similar (by two angles ∠NAF = ∠BAS and ∠ABS = ∠ANF = 90˚), from what follows FN=(AN/AB) BS=(2*a*/*a*) 2*a*=4*a*. But since the ray goes “just to the right” of point A, the reflected ray will pass “just to the left” of point F, not reflecting from the opposite wall of the hole, and will hit point M of the horizontal surface. Find the distance LM from point M from the cone axis. ∠NAF = ∠KMA (as cross-lying), then ∠BAS = ∠KMA, hence the similarity of triangles ABS and MKA (by two angles ∠BAS = ∠KMA and ∠ABS = ∠MKA = 90˚). Then KM=(KA/BS) AB=((6*a*+4*a*)/2*a*) *a*=5*a*.

$$LM = KM - LK = 5a - a = 4a$$

The ray coming through the point D, hitting the lower edge of the mirror surface, based on the equality of triangles DES and DEL (by the two sides ES = EL = 6*a*, DE – joint side, and the angle between them ∠DES = ∠DEL = 90˚), will be reflected from it at an angle to the perpendicular ∠EDL = ∠EDS and will hit the point L lying on the axis of the cone. Rays falling on the mirror surface between points A and D, reflecting from it, will hit the points of segment LM.

Rays coming from the source between points D and F will not experience reflection, but will fall directly on the surface between points H and H'. The ray coming “just to the right” of point D will hit point H of the surface. From the similarity of triangles DES and HLS (by the two angles ∠DSE = ∠HSL and ∠DES = ∠HLS = 90˚) the distance of point H from the cone axis is

$$LH = \frac{LS}{ES} ED = \frac{6a + 4a + 2a}{4a + 2a} a = 2a$$

By virtue of symmetry we can conclude that the rays falling on the right mirror surface between points N and F, reflecting, will hit the points of the segment LM' = LM. Thus, we have three overlapping segments: LM and LM' - illuminated by reflected light, HH' - illuminated by directly incident light. Since LM = LM' = 4*a* > LH = LH' = 2*a*, the segment HH' lies inside the segment M'M, and the points M and M' are the boundaries of the illuminated part of the surface. The part of the surface to the left of point G and to the right of point G' is also illuminated. That is, the shadow area will be on the segments GM' and MG'.

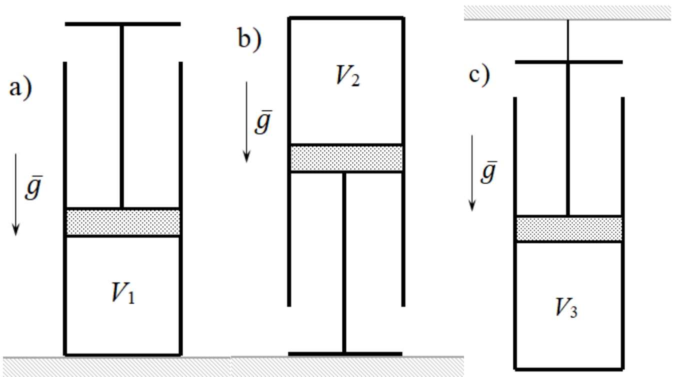
If we mentally rotate the segments GM' and MG' around the axis of the cone, they will describe a ring with inner radius r = LM = LM' = 4*a* and outer radius R = LG = LG' = 6*a*. The area of the ring can be calculated as the difference of the areas of the outer and inner circles $S = \pi\ (R^2 - r^2)$. Then the required area of the shadow is

$$S = \pi\ (R^2 - r^2) = \pi\ ((6a)^2 - (4a)^2) = 20\pi a^2 = 20\pi \left(\frac{2}{\sqrt{\pi}}\right)^2 = 80\ \text{cm}^2$$

Evaluation criteria:

The shape of the shadow is qualitatively established.	3
The outer radius of the shadow is found (with justification).	4
The inner radius of the shadow is found (with justification).	6
Analytical expression for the area of the shadow as the area of the ring is obtained.	2
Correct analytical and numerical answer is obtained.	2

Problem 5. (17 p.) The experimenters have at their disposal a cylindrical vessel of mass M with an ideal gas under a hermetic piston. The piston can move in the vessel without friction. The piston of mass m is equipped with a stem (see Fig.) with a disk at the end of the piston axis. The masses of the piston and the vessel are assumed to be unknown. If the vessel rests with its bottom on a horizontal table, the volume of gas is V_1 (a). If the structure is inverted, the volume is V_2 (b). If the system is suspended by a piston stem, the volume of gas is V_3 (c). Find the ratio m/M. The mass of the gas can be neglected compared to the masses of the piston and the vessel. The steady-state temperature of the gas is the same in all cases.



Possible solution:

The amount of gas and temperature are the same in all cases, and we write the Mendeleev-Clapeyron equation in the form:

$$p = \frac{c}{V}, \qquad c = const$$

From the balance of forces acting on the piston or vessel, respectively, find the pressure in the vessel in various cases:

$$p_1 = \frac{mg}{S} + p_a = \frac{c}{V_1}$$

$$p_2 = \frac{Mg}{S} + p_a = \frac{c}{V_2}$$

$$p_3 = p_a - \frac{Mg}{S} = \frac{c}{V_3}$$

where p_a is the atmospheric pressure, S is the area of the piston, and g is the free fall acceleration. Combining these equations we obtain the required ratio

$$\frac{2Mg}{S} = \mathrm{c}\left(\frac{1}{V_2} - \frac{1}{V_3}\right),$$

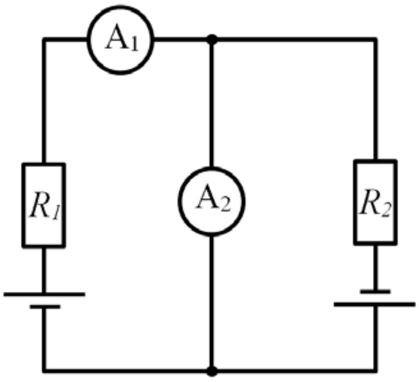
$$\frac{2mg}{S} = \mathrm{c}\left(\frac{2}{V_1} - \frac{1}{V_2} - \frac{1}{V_3}\right),$$

$$\frac{m}{M} = \frac{\left(\frac{2}{V_1} - \frac{1}{V_2} - \frac{1}{V_3}\right)}{\left(\frac{1}{V_2} - \frac{1}{V_3}\right)} = \frac{2V_2V_3 - V_1(V_3 + V_2)}{V_1(V_3 - V_2)}.$$

Evaluation criteria:

Using the Mendeleev-Clapeyron equation, pressure is related to volume in various cases.	4
Pressure is related to the mass of the piston using the balance of forces.	7
Unknown quantities are eliminated from the equations.	3
The required ratio is found.	3

Problem 6. (17 p.) The circuit shown in the figure is powered by two identical ideal batteries. At what values of resistance ratio R_1/R_2 the ammeter A_1 shows 3 times higher current value than the ammeter A_2 ? The ammeters are ideal, neglect the resistance of the wires.



Possible solution:

Denote as I_1 and I_2 the currents through ammeters, i_1 and i_2 the currents through resistors, respectively. Write the second Kirchhoff's law for the loops from the left and right of the central wire

$$i_1R_1 = I_1R_1 = \varepsilon$$

$$i_2R_2 = \varepsilon$$

The first Kirchhoff's law for a junction can be written in two ways depending on the direction of the current through the second ammeter.

$$a)\ i_2 = I_1 + I_2$$

$$b)\ i_2 = I_1 - I_2$$

Substitute the expression for the currents through the resistors and divide the equations by each other.

$$a)\ 1 = \frac{R_2}{R_1} \cdot \frac{I_1 + I_2}{I_1}$$

$$b)\ 1 = \frac{R_2}{R_1} \cdot \frac{I_1 - I_2}{I_1}$$

$$a)\ \frac{R_1}{R_2} = 1 + \frac{I_2}{I_1} = \frac{4}{3}$$

$$b)\ \frac{R_1}{R_2} = 1 - \frac{I_2}{I_1} = \frac{2}{3}$$

Evaluation criteria:

Kirchhoff's Second Law or Ohm's Law for a full circuit for two loops is written.	4
The current through the ammeters is related with the currents through the resistors.	3
Both current directions in the previous item are considered.	3
Ratio of resistances is obtained.	5
Both current directions in the previous item are considered.	2